An Experimental Investigation on Natural Frequency of Undamped Free Vibration on A Spring Mass System

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Abstract— **The natural frequency of 1-degree spring mass system can be calculated by using three well-known analytical methods (viz. Equilibrium method, Maximum energy method, and Rayleigh's method). The Equilibrium method is based the Newton's second law of motion and the D'Alembert's principle. The Maximum energy method and Rayleigh's method are based on the kinetic energy and potential energy of system. In this paper, the natural frequency of an un-damped spring mass system is calculated experimentally and compared it with that of the analytical value. The effect of mass and stiffness of system on the natural frequency is also discussed. The effect of experimental and exact (theoretical) natural frequencies is explained in detail.**

Keywords— **Stiffness, Time Period, Oscillation, universal vibration analyser**

I. Introduction

All the objects have a tendency to vibrate at a particular frequency according to its loading conditions. The swinging of a pendulum is a very good example of simple vibration. Oscillatory motion of any system is known as vibration (1, 3). Uneven distribution of forces causes vibration. The study of vibrations is necessary to understand their effects on mechanical systems. Whenever, any system displaced from its mean position by means of external work it will always try to gain its mean equilibrium condition. This external work is stored into body in the form of strained energy. As the external forced is released, the body emits that strain energy in form of kinetic energy and potential energy and try to reach its mean position this phenomenon causes vibration. The conversion of physical problem into mathematical model and the method of obtaining equation of motion are explained in subsequent section (9, 10). Equilibrium method, Maximum energy method, and Rayleigh's method are also discussed in subsequent sections.

II. Methods

The Equilibrium method

Equilibrium method is based on two methods. Namely, are (1) Newton's Second Low of Motion and (2) D'Alembert's principle. The equation of motion of a given system can be generated by using FBD.

Newton's Second Law of motion states that the accelerating force due to the mass should be equalled to the sum of external forces in the system (Fig. 1.).

$$m\ddot{x} = \sum External\ Forces \quad \text{... (1)}$$

Where, m = Mass, and $\ddot{x}$ = Acceleration

When mass 'm' is attached to a free spring, its static deflection would be say 'Δst', due to the self-weight W=mg.

$$K = \frac{W}{\Delta st} = \frac{mg}{\Delta st} \quad \text{... (2)}$$

Where, W = self-weight, and Δst = static deflection

D'Alembert's principle (4, 8) converts a dynamic problem into a statically problem by adding the inertia force. As per D'Alembert's principle, a dynamic body can be converted into a static equilibrium condition by means of considering an inertia force $(F = m\ddot{x})$ passing through the centre of gravity of the body in the direction opposite to the acceleration and would have a magnitude equal to the product of the mass and the acceleration.

$$\sum Internal\ Forcces + \sum External\ Forces = 0 \quad \text{... (3)}$$

$$-m\ddot{x} + mg - K(x + \Delta st) = 0 \quad \text{... (4)}$$

Where, x = Displacement

$$m\ddot{x} = mg - Kx - K\Delta st \quad \text{... (5)}$$

From equation (1)

$$mg = K\Delta st \quad \text{...(6)}$$

$$m\ddot{x} = -Kx \quad \text{... (7)}$$

$$\ddot{x} + \frac{K}{m}x = 0 \quad \text{... (8)}$$

The Simple Harmonic Motion,

$$\ddot{x} + \omega^2 x = 0 \quad \text{... (9)}$$

Now Comparing this with (8),

$$\omega^2 = \frac{K}{m} \quad \text{... (10)}$$

Represent the natural frequency of the system,

$$\omega_n = \sqrt{\frac{K}{m}} \quad \text{... (11)}$$

Where,

ω_n = natural frequency of the sring mass system

The time period,

$$T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{K}} \quad \text{... (12)}$$

The frequency in hertz will be,

$$f_n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad \text{... (13)}$$

Maximum energy method

The law of conservation of energy states that energy can neither be created nor destroyed, but it can only be converted from one form to another form. Using this method we can find the equation of motion (4, 8).

Let, U = Total Energy

PE = Potential Energy

KE = Kinetic Energy

$$U = PE + KE = Constant \quad \text{... (14)}$$

As discussed earlier, for a simple spring mass system, the self-weight effect can be ignores as the static deflection accounts for it.

$$KE = \frac{1}{2}m\dot{x}^2 \quad \text{... (15)}$$

Where, $\dot{x}$ = velocity of the mass.

The potential energy is stored into the spring in form of strain energy. The strain energy is given by the area under the force against deflection graph of a spring as shown (Fig. 2.)

The equation of potential energy of system can be written as,

$$PE = \frac{1}{2} \times hight \times base$$

$$= \frac{1}{2}Px$$

$$P = Kx$$

$$PE = \frac{1}{2}Kx^2 \quad \text{... (16)}$$

From equation (14), (15) and (16),

$$U = KE + PE$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}Kx^2 = constant \quad \text{... (17)}$$

Differentiating the Eq. (17) with respect to time, we get:

$$\frac{dU}{dt} = \frac{2}{2}m\dot{x}\ddot{x} + \frac{2}{2}Kx\dot{x} = 0 \quad \text{... (18)}$$

$\dot{x} \neq 0$ as the body has a defkne velocity,

We get the equation of motion as

$$m\ddot{x} + Kx = 0 \quad \text{... (19)}$$

$$\ddot{x} + \frac{K}{m}x = 0 \quad \text{... (20)}$$

Rayleigh's Method

The Rayleigh's method is based on the law of energy conservation. The method states that the maximum kinetic energy of the system is equal to the maximum potential energy (7, 8). The maximum kinetic energy occurs when the system has maximum velocity, and the maximum potential energy occurs when the system is at maximum displacement from the mean.

Let,

PE max = Maximum Potential Energy

KE max = Maximum Kinetic Energy

$$PE_{max} = KE_{max} \quad \text{... (21)}$$

As discussed earlier, for a simple mass system, the self-weight effect can be ignored as the static deflection accounts for it. Considering SHM,

$$x = X \sin \omega t \qquad \text{... (22)}$$

When $\sin \omega t = 0$ the displacement is maximum,

$$x_{max} = X \qquad \text{... (23)}$$

Differentiating,

$$\dot{x} = X\omega \cos \omega t \qquad \text{... (24)}$$

When $\cos \omega t = 1$ the velocity is maximum

$$\dot{x}_{max} = X\omega \qquad \text{... (25)}$$

From equation (16) and (23)

$$PE_{max} = \frac{1}{2} K {x^2}_{max} = \frac{1}{2} K X^2 \qquad \text{... (26)}$$

Similarly from equations (15) and (25)

$$KE_{max} = \frac{1}{2} m {\dot{x}^2}_{max} = \frac{1}{2} m X^2 \omega^2 \qquad \text{... (27)}$$

From equations (21), (26) and (27)

$$\frac{1}{2} K X^2 = \frac{1}{2} m X^2 \omega^2 \qquad \text{... (28)}$$

$$\omega_n = \omega = \sqrt{\frac{K}{m}} \qquad \text{... (29)}$$

The SHM equation,

$$\ddot{x} + \omega^2 x = 0 \qquad \text{... (30)}$$

Or

$$\ddot{x} + \frac{K}{m} x = 0 \qquad \text{... (31)}$$

III. EXPERIMENTAL SETUP

First of all fix the spring to the stud and attach the weight holder to the bottom of spring and note the initial reading. Then, attach the different-different weight to the spring and note the deflection for particular weight and find out the spring stiffness 'K'. Repeats this experiment by using different springs and note the reading for free vibration and also not the time for the 10 oscillation (Fig. 3.). The analytical natural frequency and experimental natural frequency are calculated as below. The readings are shown in Observation Table (Table 1.).

Calculations:-

m = 1 kg, δ = 0.014 m, t = 2.84 sec per 10 Oscillation, t = 0.284 sec / 1 Oscillation

$$W = mg = 1 * 9.81 = 9.81\ N$$

$$K = \frac{W}{\delta} = \frac{9.81}{0.014} = 700.71\ N/m$$

$$f_{th} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{700.71}{1}} = 4.21\ H_z$$

$$f_{exp} = \frac{1}{T_{exp}} = \frac{1}{0.284} = 3.52\ H_z$$

$$Difference = 16.18\ \%$$

m = 1.5 kg, δ = 0.021 m, t = 3.52 sec per 10 Oscillation, t = 0.352 sec / 1 Oscillation

$$W = mg = 1.5 * 9.81 = 14.71\ N$$

$$K = \frac{W}{\delta} = \frac{14.71}{0.021} = 700.71\ N/m$$

$$f_{th} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{700.71}{1.5}} = 3.44\ H_z$$

$$f_{exp} = \frac{1}{T_{exp}} = \frac{1}{0.352} = 2.84\ H_z$$

$$Difference = 17.44\ \%$$

m = 2 kg, δ = 0.028 m, t = 3.92 sec pre 10 Oscillation, t = 0.392 sec / 1 Oscillation

$$W = mg = 2 * 9.81 = 19.62\ N$$

$$f_{th} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{700.71}{2}} = 2.92\ H_z$$

$$f_{exp} = \frac{1}{T_{exp}} = \frac{1}{0.392} = 2.55\ H_z$$

$$Difference = 14.43\ \%$$

IV. RESULTS AND DISCUSSION

The natural frequency of spring mass system is analysed through experimental as well as analytical method. In the analytical method we are calculating stiffness 'K' of spring mass system by means of considering 10 independent runs as discussed earlier. Analytic natural frequency (f_{th}) and experimental natural frequency (f_{exp}) can be calculated as per above calculation and they are compare with each other. The differences between both are: 17 %, 17.3 %, and 14.71 % for spring -1, for the given weight like, 1kg, 1.5kg, and 2kg respectively and the differences are 22.46 %, 22.38 %, and 7.21 % for a spring – 2 (Table 2.).

V. Conclusion

In this paper, the experimental investigation is discussed in order to calculate the natural frequency and it is compared with experimental data. The accuracy of the experimental natural frequency is investigated using 10 independent runs of this study with different-different springs and weights. A case study of experimental setup of a spring mass system shows that the importance of stiffness, the time for considering number of oscillation, and weight. The result of the case study revealed that the use of spring mass system without proper investigation on specific detail could result in misleading the theoretical natural frequency. Therefore, experiment setup should be free from error and carried out appropriate natural frequency.

Figures

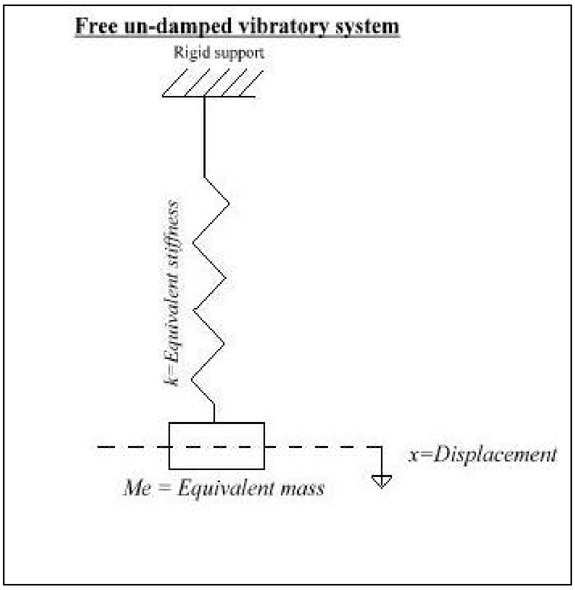

Fig.1. FBD of spring mass system

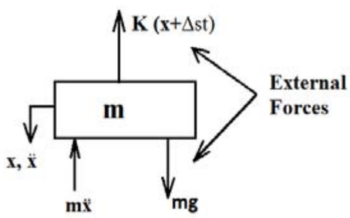

Fig. 2. Force versus Deflection of Linear Spring

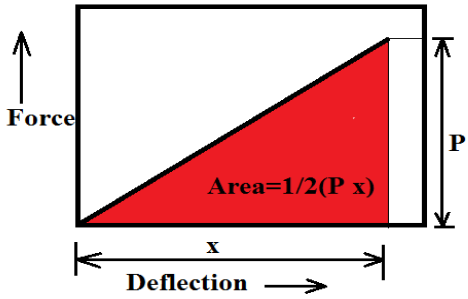

Fig.3. Experimental Setup

TABLES

Table: 1 Observation Table

Sr. No.	Spring	Attached mass m (kg)	Deflection (δ) (mm)	Time for 10 Oscillations, t (sec)
1	Spring - 1	1	25.2 - 23.8 = 1.4	1. 2.75
				2. 2.80
				3. 2.97
				Average Time: 2.84
		1.5	25.2 - 23.1 = 2.1	1. 3.48
				2. 3.56
				3. 3.52
				Average Time: 3.52
		2	25.2 - 22.4 = 2.8	1. 3.91
				2. 4.02
				3. 3.85
				Average Time: 3.92
2	Spring - 2	1	28.2 - 27.2 = 1.1	1. 2.56
				2. 2.60
				3. 2.62
				Average Time: 2.59
		1.5	28.2 - 26.6 = 1.6	1. 2.98
				2. 3.08
				3. 3.02
				Average Time: 3.02
		2	28.2 - 26.1 = 2.1	1. 3.13
				2. 3.10
				3. 3.17
				Average Time: 3.133

Table: 2 Result table

Sr. No.	Spring	Attached mass (m) (kg)	Natural Frequency (f_{exp})	Natural Frequency (f_{th})	Difference (%)
1	Spring - 1	1	3.52	4.21	17
		1.5	2.84	3.44	17.73
		2	2.55	2.99	14.71
2	Spring - 2	1	3.86	4.81	22.46
		1.5	3.05	3.94	22.38
		2	3.19	3.44	7.21

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